Exam. Code : 103202
Subject Code : 1027

B.A./B.Sc. Semester—II

MATHEMATICS

Paper–I

(Calculus and Differential Equations)

Time Allowed—3 Hours] [Maximum Marks—50

Note :– Attempt **FIVE** questions in all, selecting at least **TWO** questions from each Section.

SECTION–A

I. (a) Show that the asymptotes of the curve :

$$x^3 - xy^2 - 2xy + 2x - y - 1 = 0$$

cut the curve in at most three points which lie on line $3x - y - 1 = 0$.

(b) Show that the abscissa of the point of inflexion on the curve :

$$x = a - b\cos\theta,\ y = a\theta - b\sin\theta \text{ is } \frac{a^2 - b^2}{a}.$$ 5,5

II. (a) Show that at the point (1, –1), there is a cusp on the curve :

$$x^3 + xy^2 + y^3 - 4x^2 + y^2 + 4x + y - 1 = 0.$$

(b) Prove that for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $P=\frac{CD^3}{ab}$, where CD is the semi conjugate diameter to CP. 5,5

III. (a) Trace the curve $y^2(a+x)=x^2(3a-x)$, $a>0$.

(b) Evaluate $\int \frac{\sinh x+\cosh x}{\sinh^3 x-\cosh^3 x}dx$. 5,5

IV. (a) If $U_n=\int_0^{\frac{\pi}{4}} \tan^n dx, n>1$ show that

$$U_n+U_{n-2}=\frac{1}{n-1}\text{; deduce the value of } U_5.$$

(b) Show that $\int_0^{\pi/2} \sin^{2m}\theta \cos^{2m-1}\theta \, d\theta$

$$=\frac{(2m-2)(2m-4)--4.2}{(4m-1)(4m-3)---(2m+1)}$$, m being a positive interger > 1. 5,5

V. (a) Prove that $\int_0^{\pi} \frac{xdx}{a^2\cos^2 x+b^2\sin^2 x}=\frac{\pi^2}{2ab}$.

(b) Find the length of the arc of the parabola $x^2=4ay$ extending from the vertex to one extremity of the latus-rectum. 5,5

SECTION–B

VI. (a) Find the necessary and sufficient condition that the equation :

$Mdx + Ndy = 0$ may be exact.

(b) Solve : $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0.$ 5,5

VII. (a) Solve and examine for singular solution of the differential equation :

$x^2(y - px) = yp^2.$

(b) Find the orthogonal trajectory of the series of parabolas whose equation is $y^2 = 4ax$. 5,5

VIII. (a) Solve : $(2x-1)^3 \frac{d^3y}{dx^3} + (2x-1)\frac{dy}{dx} - 2y = x$

(b) Solve $(D^2 + a^2)\, y = \sec ax$, by method of variation of parameters. 5,5

IX. (a) Solve in series :

$$(x - x^2)\frac{d^2y}{dx^2} + (1 - 5x)\frac{dy}{dx} - 4y = 0$$

(b) Solve in series :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 1)y = 0$$ 5,5

X. (a) Solve in series Legendre's Differential Equation.

(b) Solve : $(x^3D^3 + 3x^2D^2 + xD + 1)y = x \log x$

5,5